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ESTIMATING VALUE AT RISK DURING THE STORM: A STUDY OF 25 EUROPEAN MARKETS

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Abstract

This paper investigates which approach to estimating a common risk metric - the Value at Risk (VaR)– yields optimal results in times of significant market turbulence. To this end we leverage data on 25 European stock exchanges over a 15-year period ending in December 2020. Using data on the first 14 years, we estimate the non-parametric, the parametric Gaussian and the Cornish-Fisher versions of the VaR and compare those estimates to the actual realization in the last year of the period. A number of error metrics are consulted with both the mean absolute percentage error (MAPE) and the root mean squared error (RMSE) showing that a Gaussian parametric VaR yields the most accurate approximation to the actual value. Some implications of these results are outlined.

Keywords: *Value at Risk; VaR; parametric estimation; stock markets; risk; expected loss*
JEL Codes: *G11; G32*

Introduction

Risk is a fundamental feature of all financial assets, and the analyst merely tries to get an approximate evaluation of its magnitude by estimating a number of metrics. They range from rather simple ones such as the standard deviation or the Sharpe ratio, all the way into more sophisticated ones such as Value at Risk (VaR) and Expected Tail Loss (ETL). The Value at Risk, in particular, has turned out to be a widely used metric in the realms of both financial and operational risk and has had an outsized influence on making quantitative risk management decisions. There is, however, no single way to calculate it and the resulting estimates may vary (Guharay et al., 2017). This is particularly true during periods of high market volatility such as the year 2020 which was marked by a global health pandemic and a corresponding downturn in many global economies. Some pundits have gone as far as to remind us that the VaR metric is like an airbag that works only when it is not needed, underlining the dependence of the estimate on past data and underlying

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assumptions that may not hold during a crisis. This short paper looks deeper into this question by probing which of the alternative VaR estimation methods are most accurate during a crisis and can thus be more reliably used. The next section presents a brief literature review, followed by an explication of the data and methodology used. Section four then presents the major results and implications, while section five concludes.

Essence of the Value at Risk (VaR) Metric

The Value at Risk metric is tasked with answering a simple question: assuming that asset returns are essentially a random variable, then what is the maximum loss that can be expected with a certain probability (denoted α). By means of example calculating a daily VaR of 7% for α = 95% means that a loss of no more than 7% is expected in 95% of cases. Alternatively, losses of more than 7% are to be expected in 5% of the cases, or in 1 of 20 operating days. Formally, VaR_α is defined as follows (Esterhuysen et al., 2008; Jorion, 2006):

$$VaR_\alpha(L) = \inf\{l \in \mathbb{R}: P(L > l) \leq 1 - \alpha\} \quad (1)$$

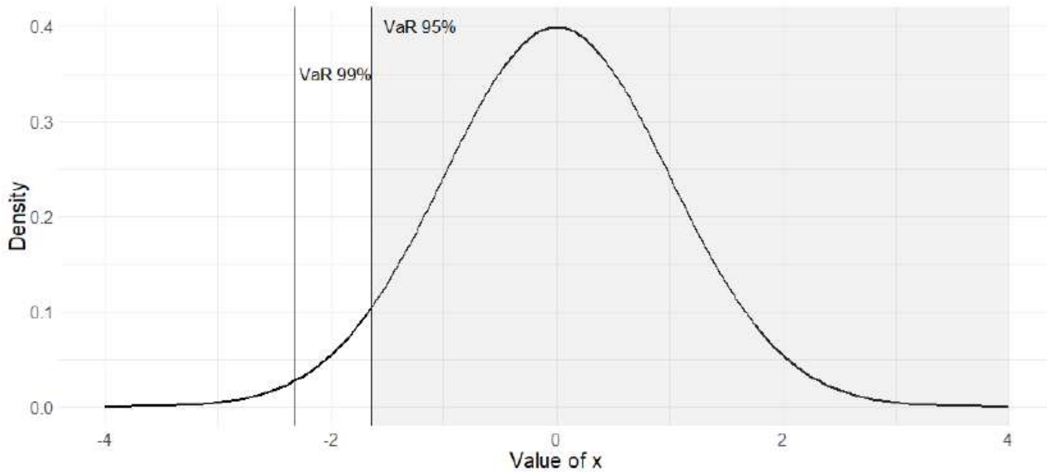
Here, the actual loss realized is denoted as l, and the expected loss as L. Should this expected loss L follow a given well-defined statistical distribution function F_L, then the VaR_α simplifies to the following expression:

$$VaR_\alpha(L) = \inf\{l \in \mathbb{R}: F_L(l) \geq \alpha\} \quad (2)$$

In practical terms, the estimation of the VaR depends on understanding the distribution and variability of asset returns of interest. The parameters of this distribution may be inferred (and mostly are) by using long historical time series or (less often) by means of simulations. Once the distribution is clear, then one easily finds the α-th percentile of the distribution, and this is the VaR. The logic behind this is schematically represented in Figure 1.

It shows a well-defined distribution function, following the Gaussian distribution with a mean of 0 and a standard deviation of 1: $x \sim N(0,1)$. The 95% VaR is at -1.64, meaning that in 95% of the cases, the losses should not exceed 1.64. Conversely, in 5% of the cases losses are likely to exceed this value. Similarly, the 99% VaR stands at about -2.33, implying that in 99% of the cases losses should not be more than 2.33, and in 1% of the cases losses are likely to exceed this threshold.

Figure no. 1 Value at Risk at 95% and 99% levels



Source: Visualization by author

The main challenge in the precise VaR estimation lies not in the calculation of percentiles but in the construction of the underlying statistical distribution based on only sampled data such as stock returns over a certain period. Overall, there have been three major groups of proposed estimation approaches: the parametric, non-parametric and simulation (or Monte Carlo methods) estimation (Guharay et al., 2017; Koike & Hofert, 2020).

The **parametric estimation** uses historical time series to infer the type of statistical distribution and fit its parameters. The most important distinguishing feature is that it assumes the type of distribution (often the Normal one), and then estimates its parameters accordingly. Correct estimation is highly dependent upon a large sample of data that includes different possible regimes of the asset dynamics. At minimum, a full economic cycle with both booms and busts should be included. On top of that the analyst may apply some correction for non-normality or skewness of data such as the Cornish-Fisher method (Kokoris et al., 2020).

The **non-parametric estimation** also uses historical data but does not depend on any assumption about the distribution of returns. Sometimes non-parametric estimation uses simulations and resampling to improve estimation precision. Most importantly, this group of approaches frees the analyst from the need to assume the shape of the distribution beforehand and instead can just let data speak for themselves. This also minimizes errors as it diminishes the cases where wrong assumptions under the VaR calculation. In addition to those obvious benefits, there is also some research showing that non-parametric methods tend to outperform parametric and semi-parametric ones (Huang et al., 2020).

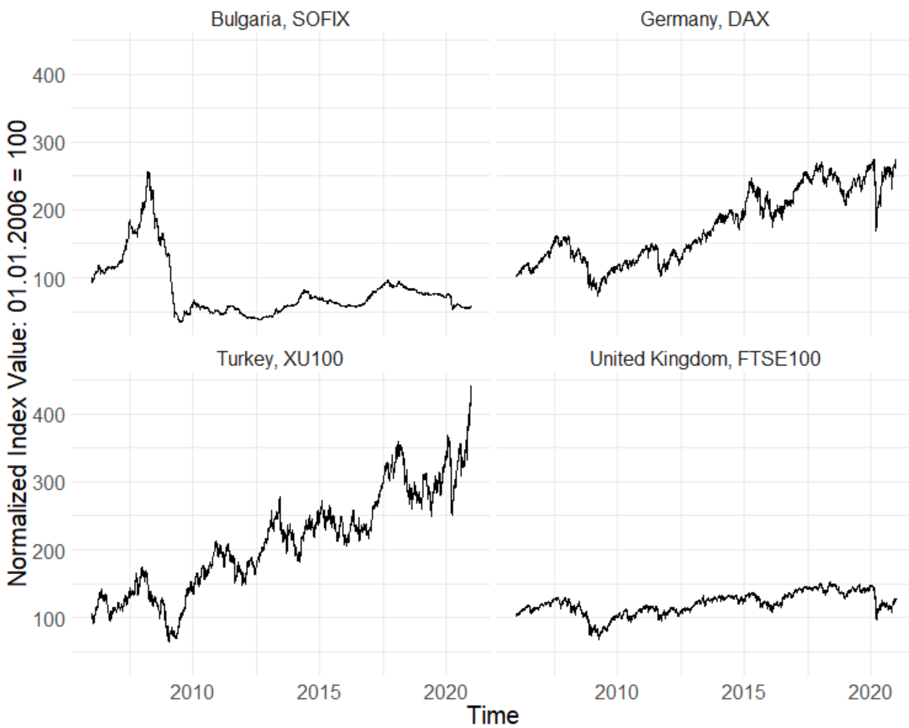
The third large group of approaches is using a **Monte Carlo-type simulation** for estimating the distribution and calculating the needed percentiles. To do this, analysts may create a small simulation that models the variable under interest and its potential drivers. Those are then parameterized and simulated by taking recourse to drawing random numbers from a set of predefined distributions for those drivers. This procedure is iterated a large number of times, the results are aggregated, and the distribution of returns is derived. In addition to using exclusively Monte Carlo methods for VaR calculation, they are also useful in complementing parametric and non-parametric estimation in cases of insufficient or incomplete data. While all three approaches to calculating the VaR are used it is not clear whether any one of them is superior in every case. This clearly put forward the challenge of choice of approach, particularly in the face of significant market volatility.

Data and Methodology

In order to compare the three main VaR calculation approaches we use extensive stock market data from 25 European stock exchanges. Those are the Euronext Lisbon, Budapest Stock Exchange, Oslo Stock Exchange, Istanbul Stock Exchange, Bulgarian Stock Exchange, Ukraine Stock Exchange, Slovakia Stock Exchange, Amsterdam Stock Exchange, Vilnius Stock Exchange, Euronext Brussels, Euronext Paris, Moscow Exchange, Bolsa de Madrid, Bucharest Stock Exchange, London Stock Exchange, FTSE 100, Helsinki Stock Exchange, Estonian Stock Exchange, Latvian Stock Exchange, Riga Stock Exchange, Stockholm Stock Exchange, Borsa Italiano, Warsaw Stock Exhange, Deustche Boerse, Athens Exchange, Prague Stock Exchange, Swiss Exchange, with their respective representative indices (see Table 1). The times series encompass 15 years, starting in January 2006 and ending in December 2020. This period is particularly fortunate as it sees both global economic expansion in the mid-2000s as well as the global financial crisis and ensued from 2008 onwards and the corresponding recovery. Most notably, the period also includes a particularly stormy year – 2020, which was characterized by a global health pandemic and very significant market volatility. All the data is procured through the public repository of Stooq and is processed in the R language for statistical programming.

European stock market could arguably not have any more diverse dynamics than they display in this period under study (see Figure 2). Some of them such as the Bulgarian market (SOFIX) have had a strong growth in the early years, only to dampen afterwards. The Germany index, DAX, has registered robust growth over the period with a clear upward trend. United Kingdom’s FTSE 100 have remained quite stable over the last 15 years, while some emerging markets such as Turkey have registered an explosive upward trajectory. Other markets also have their specifics and while it seems that al markets are moved by overall global macroeconomic risk, they also exhibit their own idiosyncratic episodes.

Figure no. 2 Four Selected Normalized European Stock Market Indices over the Period 2006-2020



Source: Stooq Database (available online at stooq.pl)

Those pronounced index dynamics make it particularly interesting to measure their Value at Risk and investigate which of the major contending approaches is most suitable for a period of extreme volatility such as the year 2020. VaR calculations are done withing the implementation proposed by Peterson et al. (2018). Initially, we select data for the period 2006 to 2020 and calculate all the relevant 95% daily VaR metrics for all markets, and then we compare them with the realized baseline 2020 daily VaR. For a more detailed description of calculation approach, please refer to Gerunov (2017).

Formally, we look not only at the absolute error rates (denoted ε) but also at the absolute percentage errors. The overall Mean Absolute Percentage Error (MAPE) gives an idea of what is the average percentage deviation of the calculated VaR metrics when compared to the baseline. We denote the percentage deviation between forecast and baselines for case *i* as p_i, over a number of forecasts, *n*. MAPE is thus defined as follows:

$$MAPE = \frac{1}{n}\sum_{i=1}^{n} 100 * |p_i| \tag{3}$$

Another, and arguably more common, metric to compare forecasts to realizations is the Root Mean Squared Error. It starts by calculating the squares of the errors, εi (difference between forecast and realization), and then takes the square root of their mean. RMSE can then be expressed as follows:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\varepsilon_i)^2} \tag{4}$$

When deciding on the best calculation approach, it is often a useful idea to consider more than one metric as their possible convergence is an indication of the robustness of results achieved. The current study thus leverages both the absolute error rate, the MAPE, and the RMSE.

Results and Discussion

Calculation results for all 25 indices are presented in Table 1. Historical calculation assumes a non-parametric estimation on the 14-year data ending in 2019, while Gaussian assumes parametric estimation, assuming normal (Gaussian) distribution of returns. The Modified estimate supposes a fat-tailed distribution of returns (as one should expect in times of high volatility) and applies the Cornish-Fisher correction. Finally, the baseline for 2020 is the calculated Value at Risk for this year without taking recourse to any assumptions, corrections, or adjustments. Daily VaRs in the European markets vary from 1.7 to around 3% meaning that in 95% of the cases this should be the maximum expected loss. In line with expectations VaRs tend to be higher for emerging and riskier markets, and especially for smaller markets. The highest VaR goes to Greece, followed by Russia, and Italy. The VaR metrics tend to be lower for the countries in the North and West of the continent.

Table no. 1 Value at Risk at 95% Calculated using Three Approaches Compared to Baseline

Country	Exchange	Index	Historical	Gaussian	Modified	Baseline 2020
Portugal	Euronext Lisbon	PSI20	2.01%	2.03%	1.89%	2.27%
Iceland	Iceland All Shares	ICEX	2.25%	2.43%	2.15%	2.87%
Hungary	Budapest Stock Exchange	BUX	2.20%	2.29%	2.27%	3.06%
Norway	Oslo Stock Exchange	OSEAX	2.55%	2.62%	2.57%	2.65%
Turkey	Istanbul Stock Exchange	XU100	1.56%	1.82%	1.78%	1.28%
Bulgaria	Bulgarian Stock Exchange	SOFIX	2.72%	3.12%	2.69%	1.95%
Ukraine	Ukraine Stock Exchange	UX	1.66%	1.83%	1.59%	1.95%
Netherlands	Amsterdam Stock Exchange	AEX	1.96%	2.08%	1.86%	3.07%
Lithuania	Vilnius Stock Exchange	OMXV	1.33%	1.64%	1.14%	0.93%
Belgium	Euronext Brussels	BEL20	1.94%	1.98%	1.84%	3.80%
France	Euronext Paris	CAC40	2.14%	2.23%	1.98%	3.54%
Russia	Moscow Exchange	MOEX	2.53%	3.12%	1.70%	2.39%
Spain	Bolsa de Madrid	IBEX	2.29%	2.38%	2.10%	3.28%
Romania	Bucharest Stock Exchange	BET	2.02%	2.36%	2.17%	2.46%
UK	London Stock Exchange, FTSE 100	UKX	1.73%	1.85%	1.65%	3.34%
Finland	Helsinki Stock Exchange	HEX	2.11%	2.16%	2.03%	3.02%
Estonia	Estonian Stock Exchange	OMXT	1.47%	1.65%	1.28%	1.11%
Latvia	Latvian Exchange, Riga All-shares Index	OMXR	1.82%	2.00%	1.46%	1.18%
Sweden	Stockholm Stock Exchange	OMXS	2.14%	2.19%	2.01%	3.15%
Italy	Borsa Italiano	FMIB	2.49%	2.56%	2.42%	3.34%
Germany	Deustche Boerse	DAX	2.10%	2.15%	1.92%	3.69%
Greece	Athens Exchange, Athens Composite	ATH	3.15%	3.24%	3.07%	4.00%
Czech R.	Prague Stock Exchange	PX	1.94%	2.20%	1.81%	2.68%
Switzerland	Swiss Exchange	SMI	1.66%	1.78%	1.60%	2.27%
Poland	Warsaw Stock Exhange	WIG20	2.21%	2.33%	2.30%	3.22%

Source: Calculations by author

Those results are unsurprising as those markets are well-established, developed, and liquid. Their institutional foundation and macroeconomic fundamentals spell a lower level of risk that is reflected in practically all metrics – from expected loss to volatility.

Table no. 2 Error Rates for Different VaR Estimates

Index	Absolute Error			Absolute Percentage Error			Root Squared Error		
	Hist	**Gauss**	**Modif**	**Hist**	**Gauss**	**Modif**	**Hist**	**Gauss**	**Modif**
PSI20	0.26%	0.24%	0.38%	12.85%	11.57%	19.81%	0.26%	1.78%	1.66%
ICEX	0.62%	0.44%	0.73%	27.67%	18.02%	33.78%	0.62%	1.81%	1.71%
BUX	0.86%	0.77%	0.79%	39.34%	33.60%	34.97%	0.86%	1.43%	1.50%
OSEAX	0.09%	0.02%	0.08%	3.59%	0.84%	3.12%	0.09%	2.53%	2.54%
XU100	0.28%	0.54%	0.49%	17.89%	29.63%	27.84%	0.28%	1.54%	1.24%
SOFIX	0.77%	1.17%	0.74%	28.17%	37.42%	27.41%	0.77%	2.35%	1.52%
UX	0.29%	0.12%	0.36%	17.24%	6.81%	22.74%	0.29%	1.54%	1.47%
AEX	1.11%	1.00%	1.21%	56.93%	47.97%	65.39%	1.11%	0.96%	0.86%
OMXV	0.40%	0.70%	0.21%	29.93%	43.03%	18.42%	0.40%	1.24%	0.44%
BEL20	1.87%	1.82%	1.96%	96.42%	91.72%	106.7%	1.87%	0.12%	0.02%
CAC40	1.40%	1.31%	1.56%	65.71%	58.89%	79.11%	1.40%	0.82%	0.66%
MOEX	0.14%	0.73%	0.69%	5.58%	23.44%	40.85%	0.14%	2.98%	0.97%
IBEX	1.00%	0.91%	1.19%	43.47%	38.15%	56.56%	1.00%	1.38%	1.19%
BET	0.44%	0.10%	0.29%	21.89%	4.18%	13.21%	0.44%	1.92%	2.07%
UKX	1.61%	1.49%	1.69%	93.57%	80.47%	102.4%	1.61%	0.24%	0.16%
HEX	0.91%	0.86%	0.99%	42.94%	39.58%	48.69%	0.91%	1.26%	1.17%
OMXT	0.36%	0.54%	0.17%	24.42%	32.89%	13.37%	0.36%	1.29%	0.74%
OMXR	0.65%	0.83%	0.28%	35.55%	41.32%	19.51%	0.65%	1.35%	0.63%
OMXS	1.01%	0.96%	1.14%	47.00%	43.66%	57.02%	1.01%	1.19%	1.05%
FMIB	0.85%	0.78%	0.92%	34.03%	30.48%	38.12%	0.85%	1.71%	1.64%
DAX	1.60%	1.54%	1.78%	76.26%	71.90%	92.77%	1.60%	0.55%	0.37%
ATH	0.85%	0.77%	0.93%	27.08%	23.72%	30.43%	0.85%	2.38%	2.30%
PX	0.73%	0.47%	0.86%	37.72%	21.52%	47.59%	0.73%	1.47%	1.34%
SMI	0.61%	0.49%	0.67%	36.75%	27.81%	41.85%	0.61%	1.17%	1.11%
WIG20	1.01%	0.89%	0.92%	45.45%	38.48%	40.07%	1.01%	1.32%	1.40%
ALL	**Average Error Rate**			**Mean Absolute % Error**			**Root Mean Squared Error**		
	0.79%	**0.78%**	**0.84%**	**38.70%**	**35.88%**	**43.27%**	**0.92%**	**0.90%**	**0.98%**

Source: Calculations by author

Another interesting insight from the table is that since estimates are based on historical data up to 2019, almost all of them underestimate the level of risk (the 95% daily VaR) that came to be realized in 2020. The baseline 2020 VaR is around 1 percentage points higher than what the historical estimates would suggest, and this result carries over even when corrections for possible fat tails of the distribution are made. What is notable for the 2020 numbers is that while the risk expectedly rises for embattled markets such as Greece, Spain, and Italy, it also does so for more stable ones such as Germany and Netherlands. Conversely, supposedly risky developing markets such as the Turkey and Russia stock exchanges register surprisingly low VaR values.

Results on the different error metrics are presented in more detail in Table 2. It seems that across the board, the best way to estimate VaR in a volatile market is to use the assumption of a normal (Gaussian distribution). The average absolute error is lowest in the Gaussian estimate (0.78%) and the non-parametric historical one (0.79%) as compared to the modified fat-tailed estimate (0.84%). Results are very similar for the MAPE metric – the estimate based on normality assumption has the lowest MAPE of 35.88%, followed by the historical (38.7%) and the modified one (43.27%). The RMSE numbers follow also reflect the superiority of the Gaussian methods. Its RMSE stands at 0.90%, closely followed by the historical one (0.92%), and more distantly – by the modified estimate with an RMSE of 0.98%. Results obtained are clear-cut, albeit a little surprising.

First, those insights add to the debate of the normality of financial returns distribution. Starting even beyond Taleb's (2007) vociferous critique, research has put doubt on the assumption that returns are normally distributed. This strand of literature continues to this day (Danielsson et al., 2013; Eom et al., 2019) and asserts that some characteristics of financial markets are better described by fat-tailed distribution. This implies that VaR metrics for stormy periods (i.e. at the tail of the distribution) should be better approximated when making a correction for the fat tail. This does not seem to be the case, and the analyst may be better served by resorting to a Gaussian distribution if in doubt.

Second, even if the correction is made, the forecast VaR tend to underestimate significantly the realized one in a volatile year. This hints that irrespective of how sophisticated a correction may be a deep crisis with significant economic repercussions has the potential to generate even larger losses than assumed and thus presents a significantly higher amount of risk. The risk and portfolio managers should then consider reserving appropriate (large) buffers instead of relying on a seemingly sophisticated corrections and assumptions that would allow them to have a very precise idea of risk.

Third, the crisis of 2020 has idiosyncratic effects on different European markets. Some safer ones such as the German one saw a significant growth in risk, while others such as the Turkish one seemed to be get calmer. Conversely, embattled markets with weaker fundamentals such as Italy, Spain, and Greece reacted cyclically. This shows that it is unwise to expect similar effects on all European markets of a given change in global macroeconomic fundamentals. Country specifics tend to dominate the resulting change in risk and the analyst should taka those into primary consideration.

Conclusion

This short paper reviewed three major approaches to calculating the Value at Risk metric and using historical data on 25 European stock market indices created forecasts for the daily VaR in 2020. This year's VaR metric was particularly challenging to estimate as the world was beset by twin health and economic crises. Unsurprisingly all three estimation methods – parametric, non-parametric, and modified Cornish-Fisher one – led to underestimates of the Value at Risk. However, the parametric method assuming a Normal distribution robustly outperforms the other two across a range of error metrics. This gives an indication that risk analysts may rely on the Gaussian assumption for their VaR calculation, at least as long as they can use long time series that encompass the full economic cycle. However, the large error rates serve to underline our uncertainty when forecasting risk and point to the necessity of provisioning for it above and beyond what models imply.

REFERENCES

Daníelsson, J., Jorgensen, B. N., Samorodnitsky, G., Sarma, M., & de Vries, C. G. (2013). Fat tails, VaR and subadditivity. *Journal of econometrics*, 172(2), 283-291.

Eom, C., Kaizoji, T., & Scalas, E. (2019). Fat tails in financial return distributions revisited: Evidence from the Korean stock market. *Physica A: Statistical Mechanics and its Applications*, 526, 121055.

Esterhuysen, J. N., Styger, P., & Van Vuuren, G. (2008). Calculating operational value-at-risk (OpVaR) in a retail bank. *South African Journal of Economic and Management Sciences*, 11(1), 1-16.

Gerunov, A. (2017). *Notes on Risk Management*. Sofia University "St. Kliment Ohridksi", ISBN: 978-954-9399-45-5.

Guharay, S., Chang, K. C., & Xu, J. (2017). Robust Estimation of Value-at-Risk through Distribution-Free and Parametric Approaches Using the Joint Severity and Frequency Model: Applications in Financial, Actuarial, and Natural Calamities Domains. *Risks*, 5(3), 41.

Jorion, P. (2006). *Value at risk: the new benchmark for managing financial risk* (Vol. 2), 3rd Ed. New York: McGraw-Hill.

Koike, T., & Hofert, M. (2020). Markov chain monte carlo methods for estimating systemic risk allocations. *Risks*, 8(1), 6.

Kokoris, A., Archontakis, F., & Grose, C. (2020). Market risk assessment. *The Journal of Risk Finance Incorporating Balance Sheet*, 21(2), 111-126.

Peterson, B.G., Carl, P., Boudt, K., Bennett, R., Ulrich, J., Zivot, E., Cornilly, D., Hung, E., Lestel, M., Balkissoon, K. & Wuertz, D. (2018). *Package "performanceanalytics"*. R Team Cooperation.

Taleb, N. N. (2007). *The black swan: The impact of the highly improbable* (Vol. 2). Random house.